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COMPACTNESS

6.1 COMPACT SPACES

178 Compact Subsets

► **EXAMPLE 108.** A subset E of X is compact if every cover of E by open subsets of X has a finite subcover.

REMARK (see 2011, p. 94). To say that a subset of a topological space is compact is to say that it is a compact space when endowed with the subspace topology. In this situation, it is often useful to extend our terminology in the following way: if X is a topological space and $A \subset X$, a collection of subsets of X whose union contains A is also called a cover of A if the subsets are open in X . We sometimes call it an open cover of A . We try to make clear in each specific situation which kind of open cover of A is meant: a collection of open subsets of a subspace union is A , or a collection of open subsets of X whose union contains A .

PROOF. The "only if" part is trivial. So we focus on the "if" part. Let \mathcal{U} be an open cover of E , i.e., $E \subset \bigcup_{U \in \mathcal{U}} U$. For every $x \in E$, there exists an open set U_x in \mathcal{U} such that $x \in U_x \cap E$. Then $\{U_x \cap E : x \in E\}$ is an open cover of E . Let $\mathcal{C} \subset \{U_x \cap E : x \in E\}$. Then there exists a finite subcover, say U_1, \dots, U_n of $\{U_x \cap E : x \in E\}$, such that $E \subset \bigcup_{i=1}^n U_i$. Hence, $E \subset \bigcup_{i=1}^n U_i \cap E$ that is, E is compact. \square

► **EXAMPLE 109.** The union of a finite collection of compact subsets of X is compact.

PROOF. Let A and B be compact, and \mathcal{U} be a family of open subsets of X which covers $A \cup B$. Then \mathcal{U} covers A and there is a finite subcover, say, U_1^A, \dots, U_m^A of A . Similarly, there is a finite subcover, say, U_1^B, \dots, U_n^B of B . But then $U_1^A, \dots, U_m^A, U_1^B, \dots, U_n^B$ is an open subcover of $A \cup B$, so $A \cup B$ is compact. \square

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